# LoRA: Low-Rank Adaptation of Large Language Models

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## Motivation

- Neural networks typically contains many dense layers with full-rank weight matrices
- Aghajanyan et al. [2021] shows that pre-trained language models have very low intrinsic dimension
- When finetune, why not make updates with low intrinsic dimension?

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## Low-Rank Parameterized Update Matrices

Given a (pre-trained) weight matrix  $W_0 \in \mathbb{R}^{d \times k}$ , LoRA constrains the update of weight with a low rank decomposition

$$W_0 + \Delta W_0 = W_0 + BA, \qquad (1)$$

where  $B \in \mathbb{R}^{d \times r}$ ,  $A \in \mathbb{R}^{r \times k}$  with  $r \ll d, k$ . To only update A, and B, during finetuning,  $W_0$  is frozen.



Figure 1: Low-Rank Adaptation (LoRA)

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## Low-Rank Parameterized Upadate Matrices

- Gaussian random for A, and zero for B. Hence at initialization  $\Delta W_0 = BA = 0$
- By increasing r, one can roughly recover full fine-tuning
- At depolyment, by computing at storing W<sub>0</sub> = W<sub>0</sub> + BA, one can eliminate additional inference latency

## Applying LoRA to Transformer

A transformer block contains two types of modules that contain dense weight matrix: attention blocks, feed forward block (MLP). In this paper, LoRA is only adapted to attention weights.



Figure 2: Transformer block

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## Self-Attention

A self-attention module contains four dense weight matrices:  $W_q, W_k, W_v, W_o$ . In this paper, LoRA is only adapted to  $W_q$  and  $W_v$ .



Figure 3: Self-attention block

Baselines

- Fine-Tuning (FT): Full fine-tuning
- Bias-only (BitFit): Fine-tuning only the biases
- Prefix-embedding tuning (PreEmbed): Prepending learned embedding of "soft prompt" to the prompt
- Prefix-layer tuning (PreLayer): Prepend learned embedding of "soft prompt" after every Transformer layer

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Adapter tuning (Adapter): Inserting adapter layers

Model & Method	# Trainable									
	Parameters	MNLI	SST-2	MRPC	CoLA	QNLI	QQP	RTE	STS-B	Avg.
RoB <sub>base</sub> (FT)*	125.0M	87.6	94.8	90.2	63.6	92.8	91.9	78.7	91.2	86.4
RoB <sub>base</sub> (BitFit)*	0.1M	84.7	93.7	92.7	62.0	91.8	84.0	81.5	90.8	85.2
RoB <sub>base</sub> (Adpt <sup>D</sup> )*	0.3M	$87.1 \pm 0.0$	$94.2_{\pm.1}$	$88.5{\scriptstyle\pm1.1}$	$60.8 \pm .4$	$93.1 \pm .1$	$90.2 \pm .0$	$71.5{\scriptstyle\pm2.7}$	$89.7_{\pm 3}$	84.4
RoB <sub>base</sub> (Adpt <sup>D</sup> )*	0.9M	$87.3_{\pm.1}$	$94.7_{\pm 3}$	$88.4_{\pm.1}$	$62.6_{\pm.9}$	$93.0_{\pm.2}$	$90.6_{\pm.0}$	$75.9_{\pm 2.2}$	$90.3_{\pm.1}$	85.4
RoB <sub>base</sub> (LoRA)	0.3M	$87.5{\scriptstyle \pm.3}$	$95.1{\scriptstyle \pm .2}$	$89.7 \scriptstyle \pm .7$	$63.4{\scriptstyle\pm1.2}$	$93.3{\scriptstyle \pm.3}$	$90.8 \scriptstyle \pm .1$	$86.6{\scriptstyle \pm.7}$	$91.5{\scriptstyle \pm .2}$	87.2
RoB <sub>large</sub> (FT)*	355.0M	90.2	96.4	90.9	68.0	94.7	92.2	86.6	92.4	88.9
RoB <sub>large</sub> (LoRA)	0.8M	$90.6_{\pm .2}$	$96.2_{\pm.5}$	$90.9_{\pm 1.2}$	$68.2_{\pm 1.9}$	$94.9_{\pm.3}$	$91.6_{\pm.1}$	$87.4_{\pm 2.5}$	$92.6_{\pm 2}$	89.0
RoB <sub>large</sub> (Adpt <sup>P</sup> )†	3.0M	90.2 <sub>±3</sub>	96.1±3	$90.2_{\pm.7}$	68.3 <sub>±1.0</sub>	$\textbf{94.8}_{\pm.2}$	$91.9_{\pm.1}$	$83.8_{\pm 2.9}$	$92.1_{\pm.7}$	88.4
RoBlarge (Adpt <sup>P</sup> ) <sup>†</sup>	0.8M	90.5±3	$96.6_{\pm 2}$	$89.7_{\pm 1.2}$	$67.8_{\pm 2.5}$	$94.8_{\pm.3}$	$91.7_{\pm.2}$	$80.1_{\pm 2.9}$	$91.9_{\pm.4}$	87.9
RoB <sub>large</sub> (Adpt <sup>H</sup> )†	6.0M	$89.9_{\pm 5}$	$96.2_{\pm 3}$	$88.7_{\pm 2.9}$	$66.5_{\pm 4.4}$	$94.7_{\pm.2}$	$92.1_{\pm.1}$	$83.4_{\pm1.1}$	$91.0_{\pm 1.7}$	87.8
RoBlarge (Adpt <sup>H</sup> ) <sup>†</sup>	0.8M	$90.3 \pm 3$	$96.3 \pm 5$	$87.7{\scriptstyle\pm1.7}$	$66.3{\scriptstyle \pm 2.0}$	$94.7 {\scriptstyle \pm.2}$	$91.5_{\pm.1}$	$72.9{\scriptstyle\pm2.9}$	$91.5 \pm 5$	86.4
RoB <sub>large</sub> (LoRA)	0.8M	$\textbf{90.6}_{\pm.2}$	$96.2_{\pm.5}$	$\textbf{90.2}_{\pm 1.0}$	$68.2{\scriptstyle\pm1.9}$	$\textbf{94.8}_{\pm.3}$	$91.6_{\pm.2}$	$85.2_{\pm 1.1}$	$92.3_{\pm 5}$	88.6
DeB <sub>XXL</sub> (FT)*	1500.0M	91.8	97.2	92.0	72.0	96.0	92.7	93.9	92.9	91.1
DeB <sub>XXL</sub> (LoRA)	4.7M	$91.9_{\pm .2}$	$96.9_{\pm2}$	$92.6_{\pm.6}$	$72.4_{\pm 1.1}$	<b>96.0</b> ±.1	$92.9_{\pm.1}$	$94.9_{\pm.4}$	$93.0_{\pm 2}$	91.3

#### Figure 4: Performance on RoBERTa, DeBERTa

Model & Method	# Trainable	E2E NLG Challenge					
	Parameters	BLEU	NIST	MET	ROUGE-L	CIDEr	
GPT-2 M (FT)*	354.92M	68.2	8.62	46.2	71.0	2.47	
GPT-2 M (Adapter <sup>L</sup> )*	0.37M	66.3	8.41	45.0	69.8	2.40	
GPT-2 M (Adapter <sup>L</sup> )*	11.09M	68.9	8.71	46.1	71.3	2.47	
GPT-2 M (Adapter <sup>H</sup> )	11.09M	$67.3_{\pm,6}$	$8.50_{\pm.07}$	$46.0_{\pm.2}$	$70.7_{\pm.2}$	$2.44_{\pm.01}$	
GPT-2 M (FT <sup>Top2</sup> )*	25.19M	68.1	8.59	46.0	70.8	2.41	
GPT-2 M (PreLayer)*	0.35M	69.7	8.81	46.1	71.4	2.49	
GPT-2 M (LoRA)	0.35M	$70.4_{\pm.1}$	$8.85_{\pm.02}$	$46.8_{\pm.2}$	$71.8_{\pm.1}$	$2.53_{\pm.02}$	
GPT-2 L (FT)*	774.03M	68.5	8.78	46.0	69.9	2.45	
GPT-2 L (Adapter <sup>L</sup> )	0.88M	$69.1_{\pm.1}$	$8.68_{\pm.03}$	$46.3 \pm .0$	$71.4_{\pm.2}$	$2.49 \scriptstyle \pm .0$	
GPT-2 L (Adapter <sup>L</sup> )	23.00M	$68.9_{\pm.3}$	$8.70_{\pm.04}$	$46.1_{\pm.1}$	$71.3_{\pm.2}$	$2.45_{\pm.02}$	
GPT-2 L (PreLayer)*	0.77M	70.3	8.85	46.2	71.7	2.47	
GPT-2 L (LoRA)	0.77M	$70.4_{\pm.1}$	$8.89_{\pm.02}$	$46.8_{\pm.2}$	$72.0_{\pm.2}$	$2.47_{\pm.02}$	

#### Figure 5: Performance on GPT-2

Model&Method	# Trainable	WikiSQL	MNLI-m	SAMSum
	Farameters	Acc. (%)	Acc. (%)	KI/K2/KL
GPT-3 (FT)	175,255.8M	73.8	89.5	52.0/28.0/44.5
GPT-3 (BitFit)	14.2M	71.3	91.0	51.3/27.4/43.5
GPT-3 (PreEmbed)	3.2M	63.1	88.6	48.3/24.2/40.5
GPT-3 (PreLayer)	20.2M	70.1	89.5	50.8/27.3/43.5
GPT-3 (Adapter <sup>H</sup> )	7.1M	71.9	89.8	53.0/28.9/44.8
GPT-3 (Adapter <sup>H</sup> )	40.1M	73.2	91.5	53.2/29.0/45.1
GPT-3 (LoRA)	4.7M	73.4	91.7	53.8/29.8/45.9
GPT-3 (LoRA)	37.7M	74.0	91.6	53.4/29.2/45.1

#### Figure 6: Performance on GPT-3

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Q. (Under the constrained budget) which weight matrices in transformer should we apply LoRA to?

A. Small r with more types of weights is better than single type of weights with a large r

	# of Trainable Parameters = 18M						4
Weight Type Rank r	$\begin{bmatrix} W_q \\ 8 \end{bmatrix}$	$\frac{W_k}{8}$	$W_v \over 8$	$\frac{W_o}{8}$	$W_q, W_k$ 4	$W_q, W_v$ 4	$W_q, W_k, W_v, W_o$ 2
WikiSQL ( $\pm 0.5\%$ ) MultiNLI ( $\pm 0.1\%$ )	70.4 91.0	70.0 90.8	73.0 91.0	73.2 91.3	71.4 91.3	<b>73.7</b> 91.3	73.7 91.7

Figure 7: Performane of different choices of weights subject to LoRA application

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Q. What is the optimal rank r?

A. LoRA works well with even with an extremely small r. Also increasing r does not guarantee better performance.

	Weight Type	r = 1	r=2	r = 4	r=8	r = 64
WikiSQL(±0.5%)	$W_q$ $W_q, W_v$ $W_q, W_w, W_q$	68.8 73.4 74.1	69.6 73.3 73.7	70.5 73.7 74.0	70.4 73.8 74.0	70.0 73.5 73.9
MultiNLI (±0.1%)	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	90.7 91.3 91.2	90.9 91.4 91.7	91.1 91.3 91.7	90.7 91.6 91.5	90.7 91.4 91.4

Figure 8: Performance of different choices of r

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Q. What is the optimal rank r?

A. LoRA works well with even with an extremely small r. Also increasing r does not guarantee better performance.

	Weight Type	r = 1	r=2	r = 4	r=8	r = 64
WikiSQL(±0.5%)	$W_q$ $W_q, W_v$ $W_q, W_w, W_q$	68.8 73.4 74.1	69.6 73.3 73.7	70.5 73.7 74.0	70.4 73.8 74.0	70.0 73.5 73.9
MultiNLI (±0.1%)	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	90.7 91.3 91.2	90.9 91.4 91.7	91.1 91.3 91.7	90.7 91.6 91.5	90.7 91.4 91.4

Figure 9: Performance of different choices of r

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- Q. Why is increasing r not effective?
- A. Top singular vector overlap significantly between r = 8 and r = 64.



Figure 10: Subspace similarity meansured by the Grassmann distance of right-singular unitary matrices for r = 8 and r = 64

 $\phi(A_{r=64},A_{r=8},i,j)$ 

Q. Does  $\Delta W$  highly correlate with W?

A. 1)  $\Delta W$  has higher correlation with W compared to a random matrix

2)  $\Delta W$  amplifies the directions that were not emphasized in W

3) Amplification magnitude is quite large

		r = 4			r = 6	4
	$\Delta W_q$	$W_q$	Random	$\Delta W_q$	$W_q$	Random
$  U^\top W_q V^\top  _F =$	0.32	21.67	0.02	1.90	37.71	0.33
$  W_q  _F = 61.95$	2	$\Delta W_q   _F =$	= 6.91	Δ	$W_q  _F =$	= 3.57

Figure 11: Performance of different choices of r

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Thank You

# Q & A

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A. Aghajanyan, S. Gupta, and L. Zettlemoyer. Intrinsic dimensionality explains the effectiveness of language model fine-tuning. Association for Computational Linguistics, 2021.